Asymmetric Alice and Bob

In this exercise the traffic flows between Alice and Bob are asymmetric, specifically, the load at Alice will be larger or equal than the load at Bob. The task is to give analytical expressions for the throughput for the case of **not** using network coding and the case of **using** network coding, see Figure 1.

a) without coding　　　　b) with coding

Figure 1: Data throughput notation in the Alice and Bob topology.

The following notation is used throughout this exercise:

Symbol	Description
i, j	Denotes a node in the scenario, $i, j = A, R, B$
X_i	The generated unit less load $X_i = 0 \ldots 1$ at node i.
$T_{i\to j}$	The throughput on the link or path from node i to node j.
$T_{i\to}$	The combined throughput on all links from node i.
$T_{\to j}$	The combined throughput on all links into node j.
C	The measured total WiFi capacity in bits/second on the shared medium.
$T_{i\to}/T_{\to i}$	loss rate at a node i

Exercise 1: Throughput without Network Coding ....

The task of this exercise is to estimate the throughputs of the given Alice and Bob topology and to fill out the following table.

Note: Each node will get a maximum of 1/3 of the capacity of the network. In the cases where packets are not dropped at a given node, the remaining nodes will share the remaining capacity equally.

	$X_B > 1/3$?	... ?	... ?	...
case	(a)	(b)	(c)	(d)
Packets dropped at	A,B,R	A,R	R	–
loss rate at R				
$T_{A\to B}$				
$T_{B\to A}$				

Note that for each case there is a condition, for example, the condition for case (a) is $X_B > 1/3$. If the condition is true, the throughputs are as given in the respective column. If the condition is not true, the condition in the next column is checked, and so on. Recall that by definition $X_A \geq X_B$. The loss rate (for a node not performing coding) is defined as $T_{i\to}/T_{\to i}$. Note that the loss rate at a node allows to compute the individual outgoing flows. This means, for instance, that the data throughput from R to B can be computed as

$$T_{R\to B} = T_{A\to R} \cdot (T_{R\to}/T_{\to R}).$$

Go through the following cases and estimate the respective throughputs $T_{A\to B}$ and $T_{B\to A}$. As input variables X_A or X_B shall be used.

(a) Packets are dropped at A, R, and B

Solution: The starting point for the analysis is the load at B, because it is smaller than the load at A. If $X_B > 1/3$, each node will get $1/3$ of the capacity. As $T_{R\rightarrow} = 1/3$, $T_{A\rightarrow R} = T_{B\rightarrow R} = 1/6$. The result for the throughputs between Alice and Bob is

$$T_{A\rightarrow B} = T_{B\rightarrow A} = 1/6 \tag{1}$$

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(b) Packets are dropped at A and R

Solution: Again, the load X_B at B is the starting point. As no packets are dropped at B,

$$T_{B\rightarrow R} = X_B. \tag{2}$$

The remaining bandwidth $1 - T_{B\rightarrow R}$ has to be shared **equally** between R and A, as both nodes require more bandwidth than available, which is expressed by the case $X_A > 0.5(1 - X_B)$. It follows:

$$T_{A\rightarrow R} = 0.5(1 - x_B) \tag{3}$$

$$T_{R\rightarrow}/T_{\rightarrow R} = \frac{0.5(1 - X_B)}{X_B + 0.5(1 - X_B)} = \cdots = \frac{1 - X_B}{1 + X_B} \tag{4}$$

$$T_{R\rightarrow A} = \mathbf{T_{B\rightarrow A}} = T_{B\rightarrow R} \cdot T_{R\rightarrow}/T_{\rightarrow R} = X_B \frac{1 - X_B}{1 + X_B} \tag{5}$$

$$T_{R\rightarrow B} = \mathbf{T_{A\rightarrow B}} = T_{A\rightarrow R} \cdot T_{R\rightarrow}/T_{\rightarrow R} = 0.5 \frac{(1 - X_B)^2}{1 + X_B} \tag{6}$$

(c) Packets are only dropped at R

Solution: If packets are only dropped at R, we can state:

$$T_{A\rightarrow R} = X_A \tag{7}$$

$$T_{B\rightarrow R} = X_B \tag{8}$$

$$T_{R\rightarrow} = 1 - X_B - X_A \tag{9}$$

$$T_{R\rightarrow}/T_{\rightarrow R} = \frac{1 - X_B - X_A}{X_B + X_A} \tag{10}$$

$$T_{R\rightarrow A} = \mathbf{T_{B\rightarrow A}} = X_B \cdot \frac{1 - X_B - X_A}{X_A + X_B} \tag{11}$$

$$T_{R\rightarrow B} = \mathbf{T_{A\rightarrow B}} = X_A \cdot \frac{1 - X_B - X_A}{X_A + X_B} \tag{12}$$

$$\tag{13}$$

Regarding the condition for case (c): If packets are dropped at the relay, we can state that $X_R > 1 - (T_{A\rightarrow R} + T_{B\rightarrow R}) = 1 - (X_A + X_B)$ (the relay requires more bandwidth than the remaining bandwidth). With $X_R = X_A + X_B$, the condition $X_A + X_B > 0.5$ follows.

(d) Packets are **not** dropped

Solution: In case no packets are dropped at any node, we can say $T_{A\rightarrow B} = X_A$ and $T_{B\rightarrow A} = X_B$. The completed table looks as follows:

	$X_B > 1/3$?	$X_A > 0.5(1 - X_B)$?	$X_A + X_B > 0.5$?	
case	(a)	(b)	(c)	(d)
Packets dropped at	A,B,R	A,R	R	–
loss rate at R	1/2	$\frac{1-X_B}{1+X_B}$	$\frac{1-X_B-X_A}{X_A+X_B}$	1
$T_{A\to B}$	1/6	$0.5\frac{(1-X_B)^2}{1+X_B}$	$X_A\frac{1-X_B-X_A}{X_A+X_B}$	X_A
$T_{B\to A}$	1/6	$X_B\frac{1-X_B}{1+X_B}$	$X_B\frac{1-X_B-X_A}{X_A+X_B}$	X_B

Exercise 2: Throughput with Network Coding ....

Now we look at the same topology and asymmetric loads using network coding. Fill out the following table regarding the cases (a), (b), and (c):

	$X_B > 1/3$?	… ?	
case	(a)	(b)	(c)
Packets dropped at	A,B	A	–
$T_{A\to B}$			
$T_{B\to A}$			

(a) Packets are dropped at A and B

Solution: In case $X_B > 1/3$,

$$T_{A\to B} = T_{B\to A} = 1/3 \tag{14}$$

It is twice than in the case without network coding, as packets are combined at the relay.

(b) Packets are only dropped at A
Explain why at R there are no packets dropped in this case.

Solution: As $X_B \leq 1/3$ (results from condition (a)), there can only packets be dropped at A or R if A requires more bandwidth than is available, similarly as in the case without coding: $X_A > 0.5(1 - X_B)$. In this case, A will get half of the remaining bandwidth:

$$T_{A\to R} = 0.5(1 - X_B) \tag{15}$$

For $T_{R\to}$ we can say that it equals the incoming flow from A:

$$T_{R\to} = T_{A\to R} = 0.5(1 - X_B) \tag{16}$$

The relay will send a combined stream of packets from A and B with the rate X_B and an uncoded stream with the rate $0.5(1-X_B) - X_B$. Thus no packets are dropped at R. The result for the flows between A and B is given as follows:

$$\mathbf{T_{B\to A}} = T_{B\to R} = X_B \tag{17}$$

$$\mathbf{T_{A\to B}} = T_{A\to R} = 0.5(1 - X_B) \tag{18}$$

(c) Packets are **not** dropped

Solution: As no packets are dropped, $T_{A\to B} = X_A$ and $T_{B\to A} = X_B$, similarly as for case (d) without network coding. The completed table is given as follows:

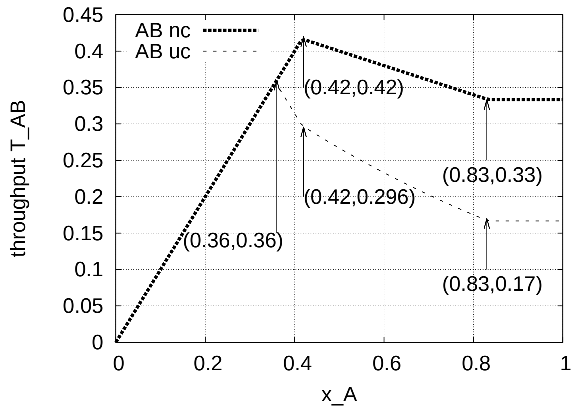
	$X_B > 1/3$?	$X_A > 0.5(1 - X_B)$?	
case	(a)	(b)	(c)
Packets are dropped at	A,B	A	–
$T_{A \to B}$	1/3	$0.5(1 - X_B)$	X_A
$T_{B \to A}$	1/3	X_B	X_B

Exercise 3: Plot of an Example Scenario with given loads ....

The ratio between the loads of A and B is given as $X_A = 2.5 \cdot X_B$, and the capacity as $C = 7.2 Mbits/s$ (you may also use a unit less medium load instead). The throughput for with and without network coding shall be compared for the flows in (a) and (b).

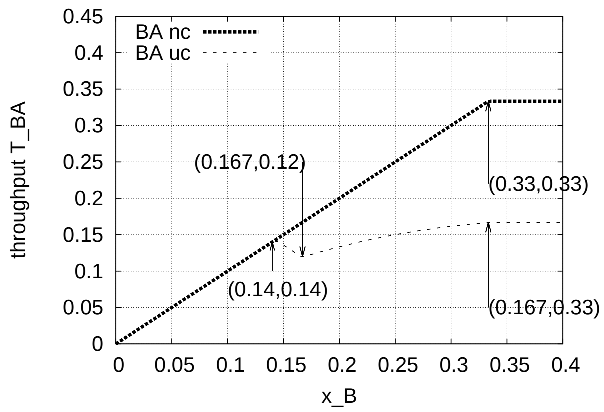
(a) Construct the plot for $T_{A \to B}$!

Solution:



(b) Construct the plot for $T_{B \to A}$!

Solution:



You can use GNUPLOT for generating the plots. A conditional plot can be generated as follows:

```
plot condition ? plot_action : alternative_plot_action
```

There can also be multiple conditions:

```
plot condition_1 ? plot_action_1 : condition_2 ? \
                   plot_action_2 : alternative_plot_action_2
```

A simple example is given as follows:

```
plot x>0.5 ? 2*x+3 : x<0.8 ? 3*x : 2
```