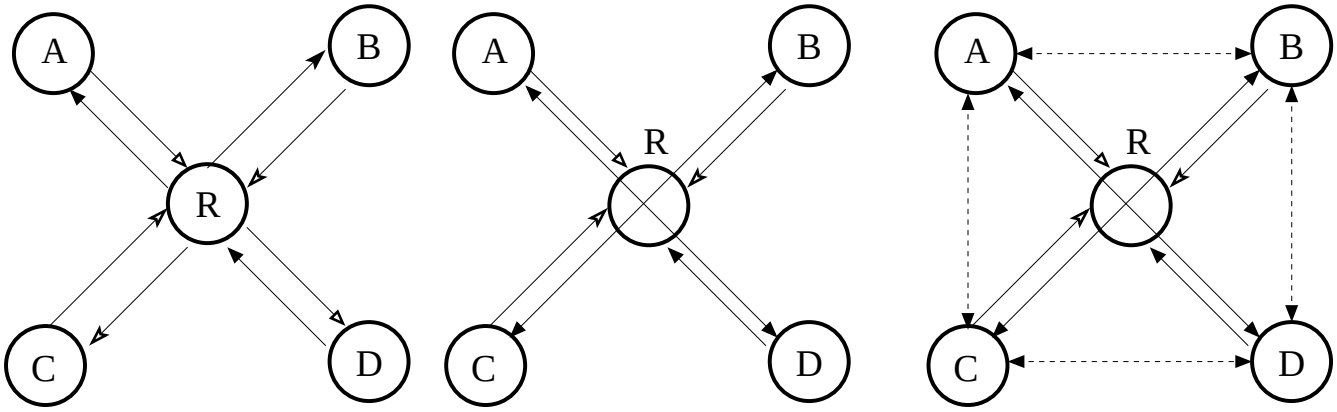
1 Bidirectional Cross

In the lecture, the plots for activity, power, and energy for the Alice and Bob topology have been discussed. Another elementary topology is the **bidirectional cross**, where we distinguish between three cases:



a) pure relaying b) network coding c) network coding with overhearing

Specifically, the cases are (a) pure relaying, where packets are not combined, (b) network coding, where two packets are combined, and (c) network coding with overhearing, where the relay combines four packets.

Exercise 1: Activities .....

Give the activities for the cases (a), (b), and (c). The possible activities to be given for each node are idle, receive, and send. The following tables are intended as a help to construct the respective activity plots.

(a) Pure Relaying
Enter the activities in the table below !

Point I: 50% load

R	r							
A	s							
B	i							
C	i							
D	i							

Relay:
___ sending
___ receiving
___ idle

Node:
___ sending
___ receiving
___ idle

Point I: 80% load

R	r				
A	s				
B	i				
C	i				
D	i				

Relay:
___ sending
___ receiving
___ idle

Node:
___ sending
___ receiving
___ idle

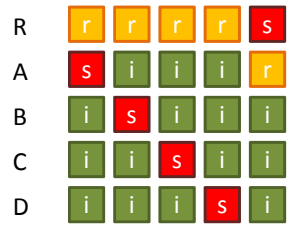
Solution:

Point I: 50% load

Relay:
4/8 sending
4/8 receiving
0/8 idle

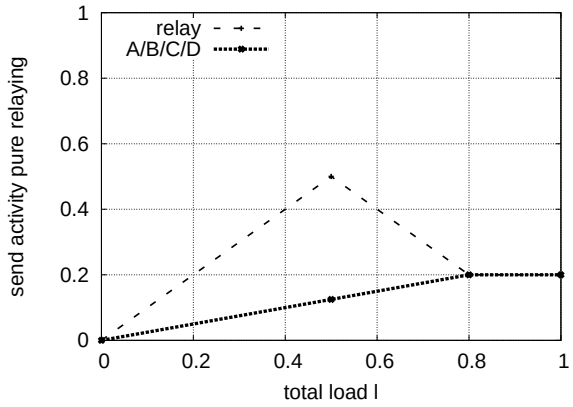
Node:
1/8 sending
1/8 receiving
6/8 idle

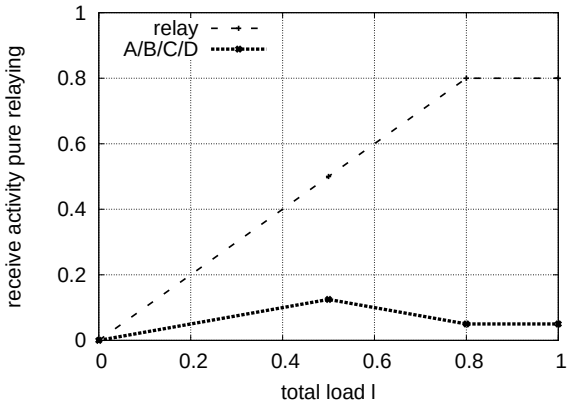
Point I: 80% load

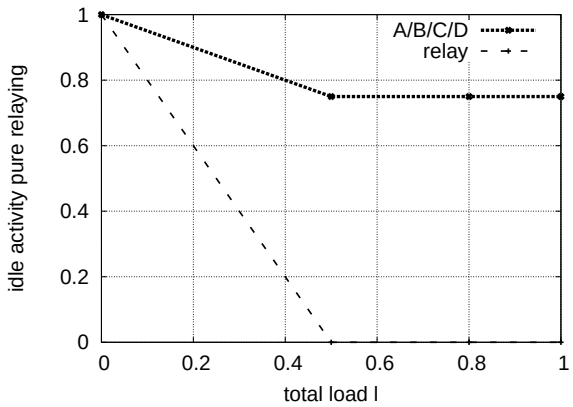


Relay:
1/5 sending
4/5 receiving
0/5 idle

Node:
4/20 sending
1/20 receiving
15/20 idle

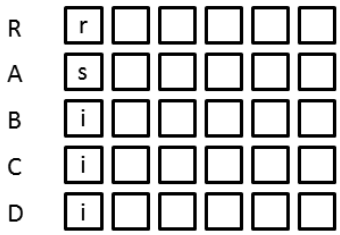






(b) Network Coding
Enter the activities in the table below !

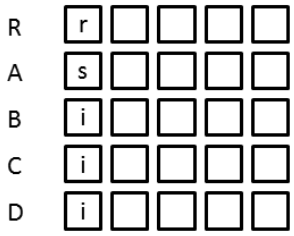
Point I: 66% load



Relay:
___ sending
___ receiving
___ idle

Node:
___ sending
___ receiving
___ idle

Point I: 80% load



Relay:
___ sending
___ receiving
___ idle

Node:
___ sending
___ receiving
___ idle

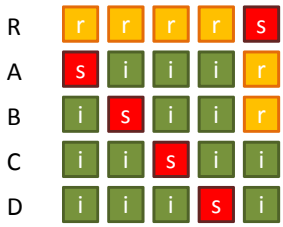
Solution:

Point I: 66% load

Relay:
2/6 sending
4/6 receiving
0/6 idle

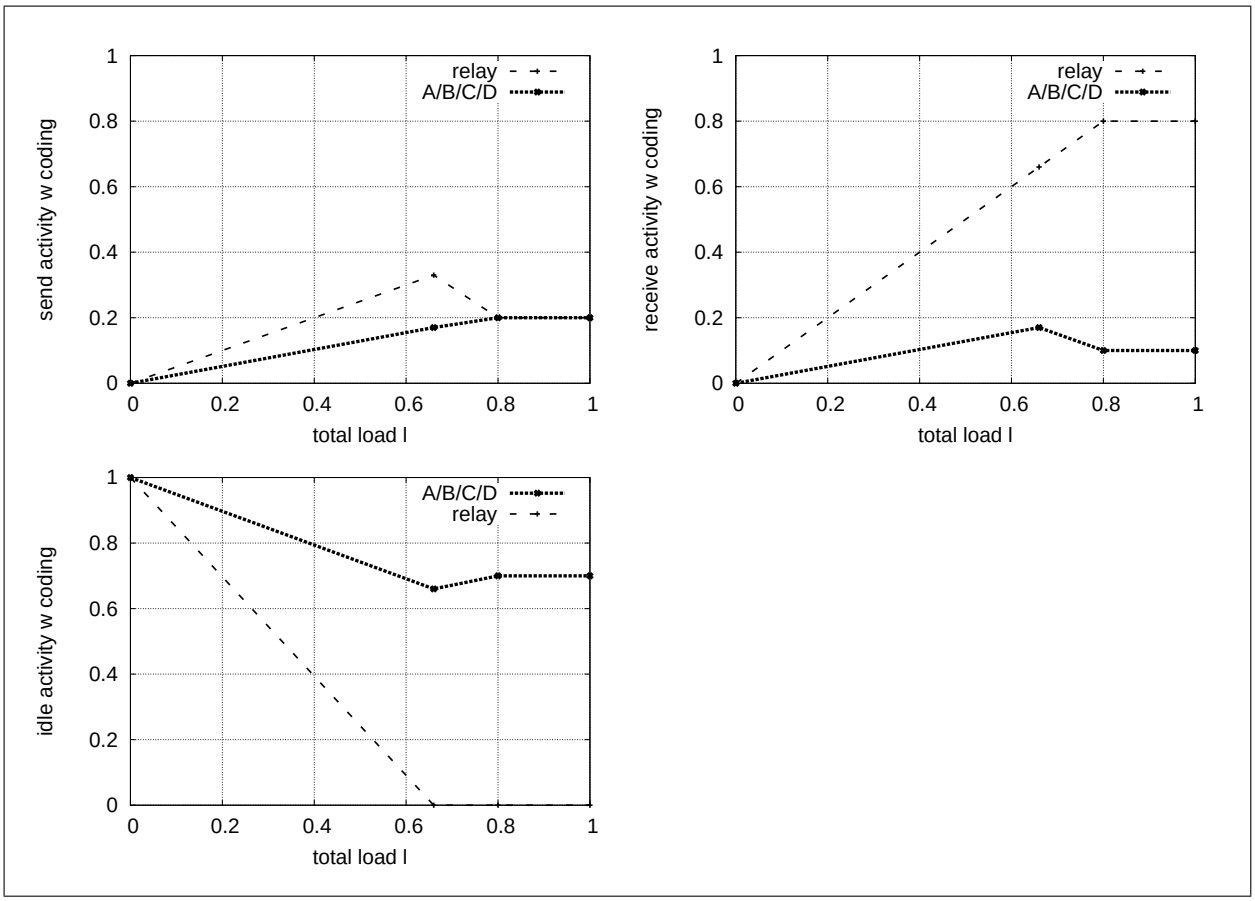
Node:
1/6 sending
1/6 receiving
4/6 idle

Point I: 80% load



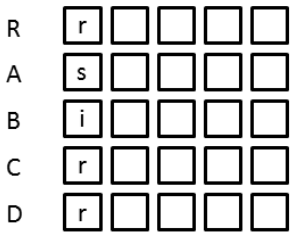
Relay:
1/5 sending
4/5 receiving
0/5 idle

Node:
4/20 sending
2/20 receiving
14/20 idle



(c) Network Coding with Overhearing
Enter the activities in the table below !

Point II: 80% load

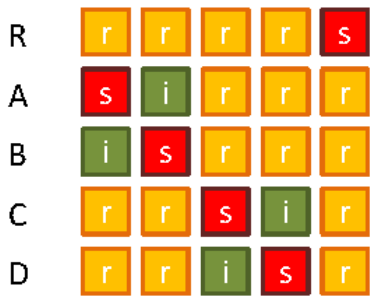


Relay:
___ sending
___ receiving
___ idle

Node:
___ sending
___ receiving
___ idle

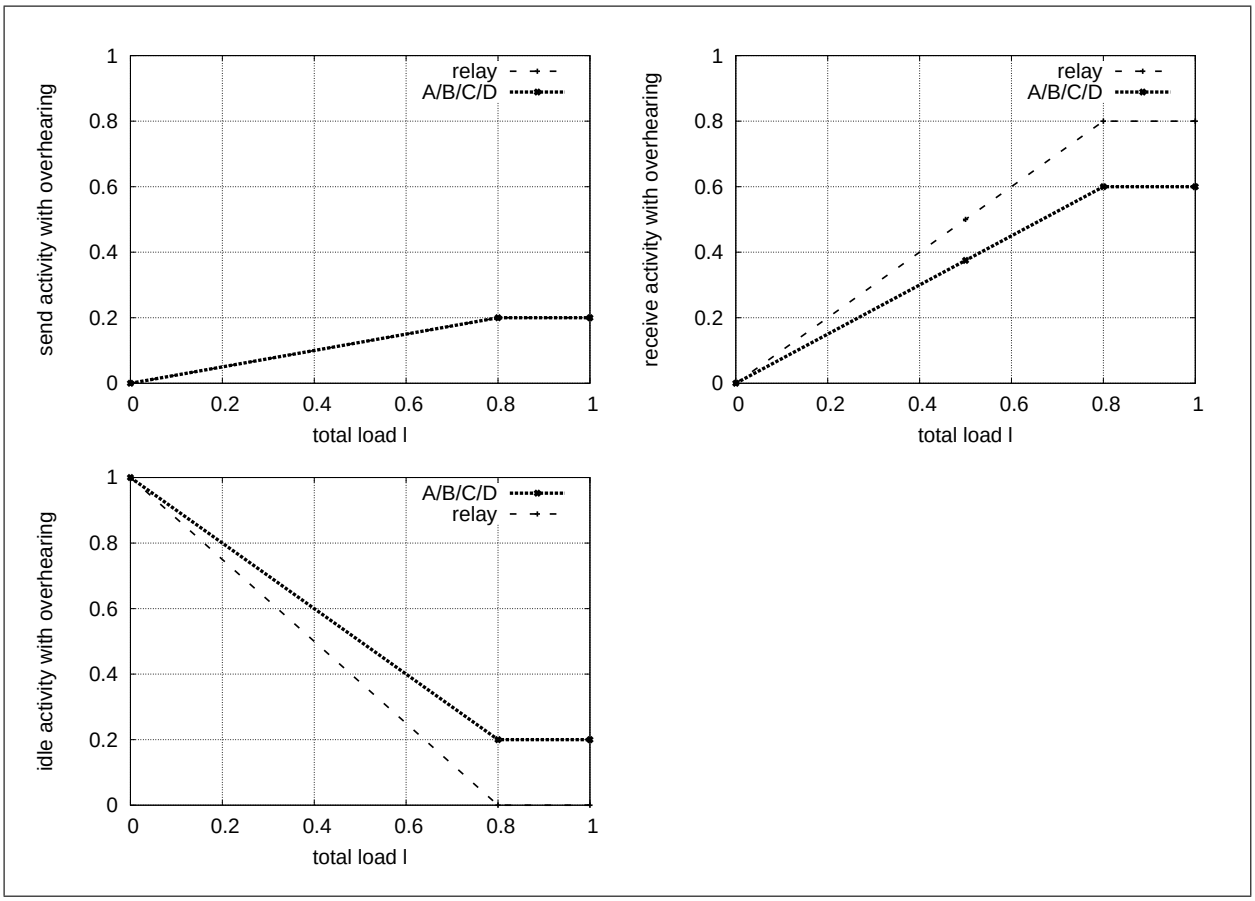
Solution:

Point II: 80% load



Relay:
1/5 sending
4/5 receiving
0/5 idle

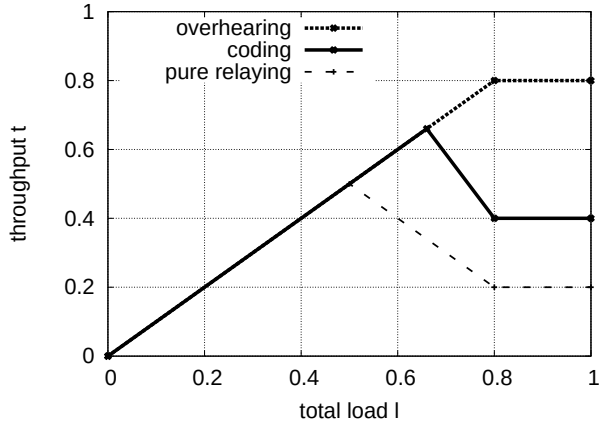
Node:
1/5 sending
3/5 receiving
1/5 idle



Exercise 2: Throughput .....

Give the total **throughput** for the three cases (a), (b), and (c), including the plot and the analytical expressions.

Solution:



Throughput for pure relaying:

$$t = \begin{cases} l & \text{if } 0 < l \leq 0.5 \\ -1 \cdot l + 1 & \text{if } 0.5 < l \leq 0.8 \\ 0.2 & \text{otherwise} \end{cases} \quad (1)$$

Throughput for network coding (without overhearing):

$$t = \begin{cases} l & \text{if } 0 < l \leq 2/3 \\ -2 \cdot l + 2 & \text{if } 2/3 < l \leq 0.8 \\ 0.4 & \text{otherwise} \end{cases} \quad (2)$$

Throughput for overhearing:

$$t = \begin{cases} l & \text{if } 0 < l < 0.8 \\ 0.8 & \text{otherwise} \end{cases} \tag{3}$$

Exercise 3: System Power

Give the **power** consumed by the complete system (including all nodes and the relay) for the three cases (a), (b), and (c). The power consumption for the states *send*, *receive*, and *idle* has been measured as $P_s = 3.48\ W$, $P_r = 3.24\ W$, and $P_i = 2.94\ W$, respectively. Give the plot and the analytical expressions.

Solution: The general formula is given as follows:

$$P(l) = P_s \cdot \alpha_s(l) + P_r \cdot \alpha_r(l) + P_i \cdot \alpha_i(l) \tag{4}$$

Now we calculate from the activity plots the striking points.
For pure relaying:

$$P(l = 0) = P_i \cdot 5 = 14.7\ W \tag{5}$$
$$P(l = 0.5) = P_s(0.125 \cdot 4 + 0.5) + P_r(0.125 \cdot 4 + 0.5) + P_i(0.75 \cdot 4) = 15.54\ W \tag{6}$$
$$P(l = 0.8) = P(l = 1) = P_s \cdot 0.2 \cdot 5 + P_r(0.05 \cdot 4 + 0.8) + P_i \cdot 0.75 \cdot 4 = 15.54\ W \tag{7}$$

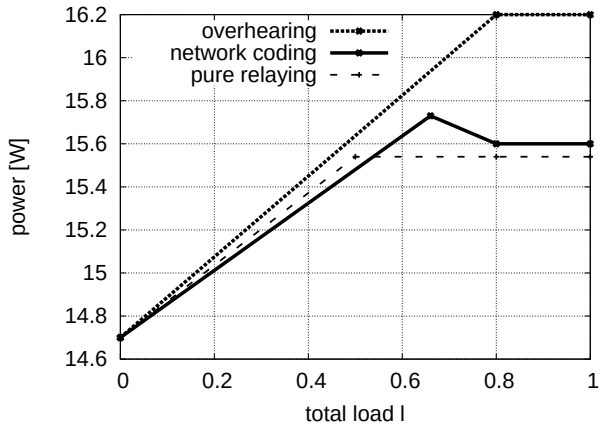
For network coding (without overhearing):

$$P(l = 0) = P_i \cdot 5 = 14.7\ W \tag{8}$$
$$P(l = 0.66) = P_s(0.17 \cdot 4 + 1/3) + P_r(0.17 \cdot 4 + 2/3) + P_i(2/3 \cdot 4) = 15.73\ W \tag{9}$$
$$P(l = 0.8) = P(l = 1) = P_s(0.2 \cdot 5) + P_r(0.1 \cdot 4 + 0.8) + P_i(0.7 \cdot 4) = 15.6\ W \tag{10}$$

For overhearing:

$$P(l = 0) = P_i \cdot 5 = 14.7\ W \tag{11}$$
$$P(l = 0.8) = P(l = 1) = P_s \cdot 0.2 \cdot 5 + P_r(0.6 \cdot 4 + 0.8) + P_i \cdot 0.2 \cdot 4 = 16.2\ W \tag{12}$$

From the points, the plot can be constructed by connecting the lines:



Now we estimate the analytical expressions from the plot.
Power for **pure relaying**:

$$P = \begin{cases} (\frac{15.54-14.7}{0.5} l)\ W + 14.7\ W = (\mathbf{1.68 \cdot l + 14.7})\ W & \text{if } 0 < l < 0.5 \\ \mathbf{15.54}\ W & \text{otherwise} \end{cases} \tag{13}$$

Power for **network coding**:

$$P = \begin{cases} (\frac{15.73-14.7}{0.66} l)\ W + 14.7\ W = (\mathbf{1.56 \cdot l + 14.7})\ W & \text{if } 0 < l \leq 2/3 \\ (-\frac{15.73-15.6}{0.8-0.66} l)\ W + 16.34\ W = (\mathbf{-0.93 \cdot l + 16.34})\ W & \text{if } 2/3 < l \leq 0.8 \\ \mathbf{15.6}\ W & \text{otherwise} \end{cases} \quad (14)$$

Power for **overhearing**:

$$P = \begin{cases} \frac{16.2-14.7}{0.8} l\ W + 14.7\ W = (\mathbf{1.875 \cdot l + 14.7})\ W & \text{if } 0 < l < 0.8 \\ \mathbf{16.2}\ W & \text{otherwise} \end{cases} \quad (15)$$

Exercise 4: Energy per Bit .....

Give the **energy** per bit for the three cases (a), (b), and (c). Use the analytical expressions for the throughput and the power. The network capacity (load $l = 1$) is estimated as $C = 4.9$ Mbit/s.

Solution: Energy=Power / Throughput.
Energy for **pure relaying**:

$$E = \begin{cases} (1.68l + 14.7)/(l \cdot 4.9 \cdot 10^6) J/bit = (\mathbf{0.3429 + 3/l}) 10^{-6} J/bit & \text{if } 0 < l \leq 0.5 \\ 15.54/((-1 \cdot l + 1) \cdot 4.9 \cdot 10^6) J/bit = \mathbf{3.17 \cdot 1/(1 - l)} 10^{-6} J/bit & \text{if } 0.5 < l \leq 0.8 \\ 15.54/(0.2 \cdot 4.9 \cdot 10^6) J/bit = \mathbf{15.86} \cdot 10^{-6} J/bit & \text{otherwise} \end{cases} \quad (16)$$

Energy with **network coding**:

$$E = \begin{cases} (1.56l + 14.7)/(l \cdot 4.9 \cdot 10^6) J/bit = (\mathbf{0.318 + 3/l}) 10^{-6} J/bit & \text{if } 0 < l \leq 2/3 \\ (-0.93l + 16.35)/((-2 \cdot l + 2) \cdot 4.9 \cdot 10^6) J/bit = (\mathbf{0.095\ l - 1.67})/(\mathbf{l - 1}) 10^{-6} J/bit & \text{if } 2/3 < l \leq 0.8 \\ 15.6/(0.4 \cdot 4.9 \cdot 10^6) J/bit = \mathbf{7.96} \cdot 10^{-6} J/bit & \text{otherwise} \end{cases} \quad (17)$$

Energy with **overhearing**:

$$E = \begin{cases} (1.875 \cdot l + 14.7)/(l \cdot 4.9 \cdot 10^6) J/bit = (\mathbf{0.38 + 3/l}) 10^{-6} J/bit & \text{if } 0 < l \leq 0.8 \\ 16.2/(0.8 \cdot 4.9 \cdot 10^6) J/bit = \mathbf{4.13} \cdot 10^{-6} J/bit & \text{otherwise} \end{cases} \quad (18)$$