MULTI–HOP CONNECTIVITY AND BANDWIDTH CALCULATION OF ARBITRARY NETWORKS

Multi–hop networks have gained a lot of interest in the last years even though their application and strength was already shown decades before in military environments. The reason for this in civil communication is based on the capability of multi hop networks to extend the coverage of existing wireless networks. The scope of this manual is to show my students the calculation of quality of service dependent parameters such as the hop distance and the bandwidth.

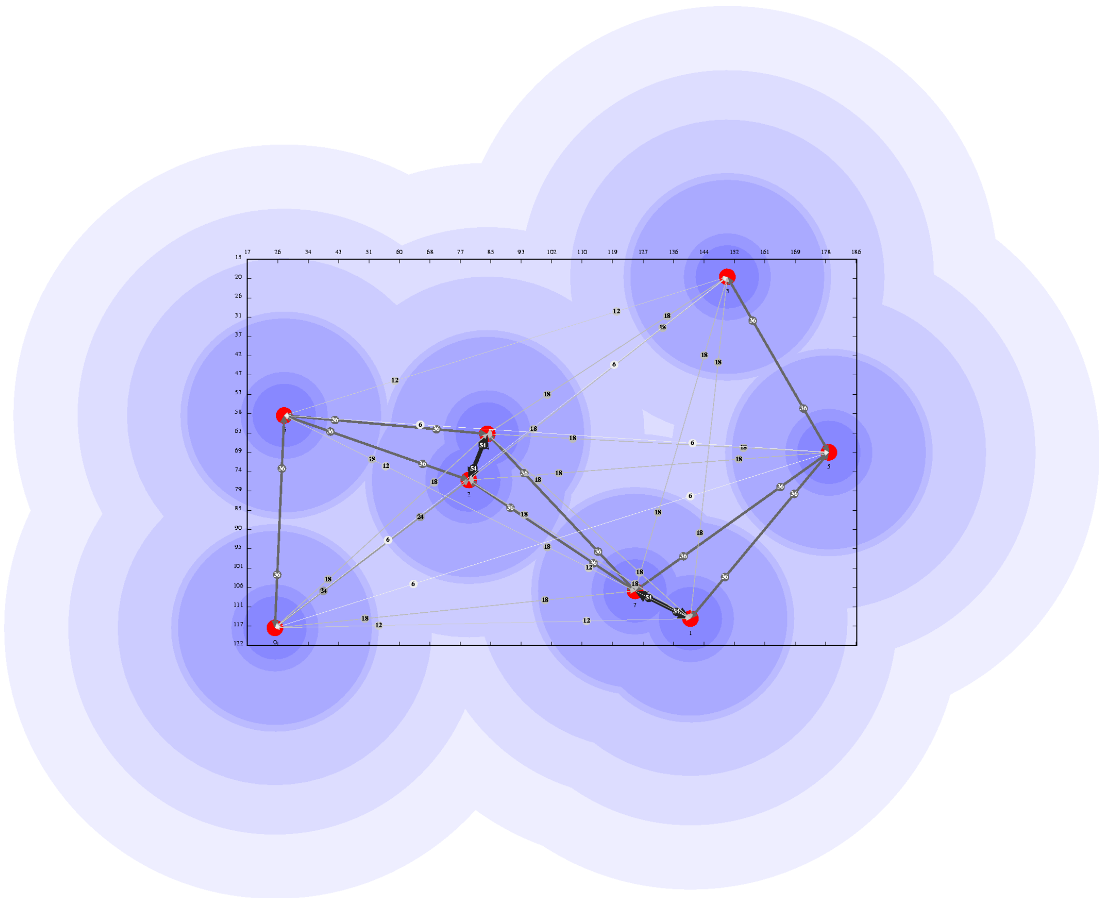

Figure 1: Topology of a Multi–Node Network with IEEE802.11a based bandwidth limitation caused by rate adaption..

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1 Preamble

Here and in my lectures we refer to multi hop networks. In the literature these type of networks are also referred to as ad hoc networks. The term ad hoc is used in different ways: 1.) to describe multi hop networks and 2.) two entities that can communicate without any administration overhead in a plug&play manner. Therefore we use the term multi–hop.

2 Multi–Hop Connectivity Approach

The following exercise shows how to calculate the multi–hop connectivity of arbitrary networks. Let us define the adjacency (single–hop connectivity) matrix $A = \{a_{ij}\}$ for a multi node network. As given in Equation 1 the Matrix A consists out of the two symbols `0` and `1`. In case a_{ij} is `0`, there is not direct connection between wireless node i and j. Otherwise a_{ij} is set to `1`.

$$a_{ij} = \begin{cases} 0 & : \ single\ hop\ i \rightarrow j\ does\ not\ exist \\ 1 & : \ single\ hop\ i \rightarrow j\ exists \end{cases} \tag{1}$$

In case we are interested not only in a direct, but also in a multi hop connection, we have to calculate the multi–hop connectivity matrix C. The following steps have to be done to calculate the matrix C.

1. INITIALIZATION: we define $C^1 = A^1$, a third intermediate matrix B^1=0 m=2
2. START: set $A^{(m)} = C^{(m-1)} \cdot C^{(m-1)}$
3. calculate $B^{(m)}$
4. if ($\forall$ i,j $b_{ij} = 0$) STOP
5. recalculate $C^{(m)} = C^{(m-1)} + B^{(m)}$
6. m++
7. GOTO START

The only missing item is how to calculate matrix $B = \{b_{ij}\}$

$$b_{ij}^{(m)} = \begin{cases} 0 & : \ i = j \\ 0 & : \ c_{ij}^{(m-1)} > 0 \\ min(c_{ik}^{(m-1)} + c_{kj}^{(m-1)}) & : \ \sum_{k-1}^{J} c_{ik}^{(m-1)} \cdot c_{kj}^{(m-1)} > 0\ when\ i \neq j\ and\ c_{ij}^{(m-1)} = 0 \\ 0 & : \ otherwise \end{cases} \tag{2}$$

In the following we want to apply this algorithm to specific network topologies.

3 Multi–Hop Bandwidth Approach

The following exercise shows how to calculate the multi–hop bandwidth of arbitrary networks. Let us define the adjacency (single–hop connectivity) matrix $R = \{r_{ij}\}$ for a multi node network. As given in Equation 3 the Matrix R consists out of the multiple symbols. In case r_{ij} is `0`, there is not direct connection between wireless node i and j. Otherwise r_{ij} equals to a Rate.

$$r_{ij} = \begin{cases} 0 & : \; single\ hop\ i \leftrightarrow j\ does\ not\ exist \rightarrow d_{i,j} > D_{MAX} \\ 6 & : \; single\ hop\ i \leftrightarrow j\ exists \rightarrow D_{12} \leq d_{i,j} < D_{6} \\ 12 & : \; single\ hop\ i \leftrightarrow j\ exists \rightarrow D_{18} \leq d_{i,j} < D_{12} \\ 18 & : \; single\ hop\ i \leftrightarrow j\ exists \rightarrow D_{24} \leq d_{i,j} < D_{18} \\ 24 & : \; single\ hop\ i \leftrightarrow j\ exists \rightarrow D_{36} \leq d_{i,j} < D_{24} \\ 36 & : \; single\ hop\ i \leftrightarrow j\ exists \rightarrow D_{48} \leq d_{i,j} < D_{36} \\ 48 & : \; single\ hop\ i \leftrightarrow j\ exists \rightarrow D_{54} \leq d_{i,j} < D_{48} \\ 54 & : \; single\ hop\ i \leftrightarrow j\ exists \rightarrow d_{i,j} < D_{54} \end{cases} \quad (3)$$

In case we are interested not only in the bandwidth between neighboring nodes, but also in a multi hop bandwidth, we have to calculate the multi–hop bandwidth matrix T. The following steps have to be done to calculate the matrix T.

1. INITIALIZATION: we define $T^1 = R^1$, a third intermediate matrix S^1=0 m=2
2. START: set $R^{(m)} = T^{(m-1)} \cdot T^{(m-1)}$
3. calculate $S^{(m)}$
4. if ($\forall$ i,j $s_{ij} = 0$) STOP
5. recalculate $T^{(m)} = T^{(m-1)} + S^{(m)}$
6. m++
7. GOTO START

The only missing item is how to calculate matrix $S = \{s_{ij}\}$

$$s_{ij}^{(m)} = \begin{cases} 0 & : \; i = j \\ 0 & : \; t_{ij}^{(m-1)} > 0 \\ min(t_{ik}^{(m-1)}, t_{kj}^{(m-1)})\ while\ min(t_{ik}^{(m-1)} + t_{kj}^{(m-1)}) & : \; \sum_{k-1}^{J} t_{ik}^{(m-1)} \cdot t_{kj}^{(m-1)} > 0\ when\ i \neq j\ and\ t_{ij}^{(m-} \\ 0 & : \; otherwise \end{cases} \quad (4)$$

In the following we want to apply this algorithm to specific network topologies.

4 Example: The Chain Topology

In Figure 2 a chain topology is given:

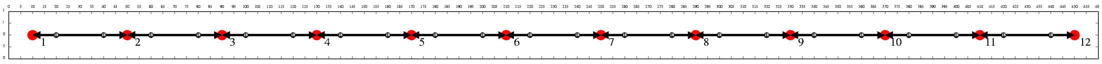

Figure 2: Topology of a Multi–Node Network.

The adjacency (single–hop connectivity) matrix A, the auxiliary matrix B, and the connectivity Matrix C of the multi–node network are given in the following:

1. Round

$$A^{(1)} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

$$B^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(1)} = \begin{pmatrix} 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$$

2. Round

$$A^{(2)} = C^{(1)} \cdot C^{(1)} = \begin{pmatrix} 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$B^{(2)} = \begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \end{pmatrix}$$

$$C^{(2)} = C^{(1)} + B^{(2)} = \begin{pmatrix} 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \end{pmatrix}$$

3. Round

$$A^{(3)} = C^{(2)} \cdot C^{(2)} = \begin{pmatrix} 5 & 2 & 1 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 6 & 4 & 1 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 & 0 & 0 \\ 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 & 0 \\ 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 \\ 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 \\ 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 \\ 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 \\ 0 & 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 4 & 1 & 4 & 6 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 4 & 1 & 2 & 5 \end{pmatrix}$$

$$B^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 0 \\ 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 \\ 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(3)} = C^{(2)} + B^{(3)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 0 & 0 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 0 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 0 & 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

4. Round

$$A^{(4)} = C^{(3)} \cdot C^{(3)} = \begin{pmatrix} 30 & 20 & 12 & 8 & 10 & 20 & 25 & 24 & 16 & 0 & 0 & 0 \\ 20 & 31 & 22 & 15 & 12 & 10 & 20 & 25 & 24 & 16 & 0 & 0 \\ 12 & 22 & 35 & 28 & 23 & 12 & 10 & 20 & 25 & 24 & 16 & 0 \\ 8 & 15 & 28 & 44 & 40 & 23 & 12 & 10 & 20 & 25 & 24 & 16 \\ 10 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 10 & 20 & 25 & 24 \\ 20 & 10 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 10 & 20 & 25 \\ 25 & 20 & 10 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 10 & 20 \\ 24 & 25 & 20 & 10 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 10 \\ 16 & 24 & 25 & 20 & 10 & 12 & 23 & 40 & 44 & 28 & 15 & 8 \\ 0 & 16 & 24 & 25 & 20 & 10 & 12 & 23 & 28 & 35 & 22 & 12 \\ 0 & 0 & 16 & 24 & 25 & 20 & 10 & 12 & 15 & 22 & 31 & 20 \\ 0 & 0 & 0 & 16 & 24 & 25 & 20 & 10 & 8 & 12 & 20 & 30 \end{pmatrix}$$

$$B^{(4)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 5 & 6 & 7 & 8 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 7 & 8 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 7 & 8 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 7 & 8 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 7 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 \\ 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 7 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 8 & 7 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 8 & 7 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 8 & 7 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 8 & 7 & 6 & 5 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(4)} = C^{(3)} + B^{(4)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 & 0 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 0 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 0 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

5. Round

$$A^{(5)} = C^{(4)} \cdot C^{(4)} = \begin{pmatrix} 204 & 168 & 134 & 104 & 80 & 64 & 58 & 64 & 84 & 120 & 147 & 164 \\ 168 & 205 & 170 & 137 & 108 & 85 & 70 & 65 & 72 & 84 & 120 & 147 \\ 134 & 170 & 209 & 176 & 145 & 118 & 97 & 84 & 81 & 72 & 84 & 120 \\ 104 & 137 & 176 & 218 & 188 & 160 & 136 & 118 & 108 & 81 & 72 & 84 \\ 80 & 108 & 145 & 188 & 170 & 152 & 136 & 124 & 118 & 84 & 65 & 64 \\ 64 & 85 & 118 & 160 & 152 & 146 & 140 & 136 & 136 & 97 & 70 & 58 \\ 58 & 70 & 97 & 136 & 136 & 140 & 146 & 152 & 160 & 118 & 85 & 64 \\ 64 & 65 & 84 & 118 & 124 & 136 & 152 & 170 & 188 & 145 & 108 & 80 \\ 84 & 72 & 81 & 108 & 118 & 136 & 160 & 188 & 218 & 176 & 137 & 104 \\ 120 & 84 & 72 & 81 & 84 & 97 & 118 & 145 & 176 & 209 & 170 & 134 \\ 147 & 120 & 84 & 72 & 65 & 70 & 85 & 108 & 137 & 170 & 205 & 168 \\ 164 & 147 & 120 & 84 & 64 & 58 & 64 & 80 & 104 & 134 & 168 & 204 \end{pmatrix}$$

$$B^{(5)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 10 & 11 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 & 10 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 9 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 10 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 11 & 10 & 9 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(5)} = C^{(4)} + B^{(5)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 & 11 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 7 \\ 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 \\ 11 & 10 & 9 & 8 & 7 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

An interesting value for a multi–hop network is the number of hops that are needed to send information from one node to another. Obviously, the number of hops depends on the chosen nodes. The maximum number of hops is N-1. The calculation of the mean number of hops $\overline{H}$ is done by summing up all hops that are need for a communication between two nodes given by matrix C and divide the sum by the overall number of all possible connections (given in Equation 5).

$$all = N \cdot (N-1) \tag{5}$$

To calculate the number of hops we see that for the chain topology the connectivity matrix C is symmetric. Furthermore, the first diagonal is set with zeros, the secondary diagonal with ones and so on. This leads us to Equation 11:

$$\begin{aligned}
\overline{H} &= 2 \cdot \frac{1}{N(N-1)} \sum_{k=1}^{N-1} k(N-k) && (6)\\
&= 2 \cdot \frac{1}{N(N-1)} \left(N \cdot \sum_{k=1}^{N-1} k - \sum_{k=1}^{N-1} k^2 \right) && (7)\\
& && (8)
\end{aligned}$$

Note,

$$\sum_{k=1}^{N} k = \frac{N(N+1)}{2} \tag{9}$$

and

$$\sum_{k=1}^{N} k^2 = \frac{N(N+1)(2N+1)}{6}. \tag{10}$$

With Equation 28 and 10 (please note the index change), Equation 11 simplifies to

$$\begin{aligned}
\overline{H} &= 2 \cdot \frac{1}{N(N-1)} \left(\frac{(N-1)N}{2} - \frac{(N-1)(N)(2(N-1)+1)}{6} \right) && (11)\\
&= 2 \cdot \frac{1}{N(N-1)} \left(N \cdot \frac{(N-1)N}{2} - \frac{(N-1)(N)(2(N-1)+1)}{6} \right) && (12)\\
&= N - \frac{(2N-1)}{3} && (13)\\
&= \frac{N+1}{3} && (14)\\
& && (15)
\end{aligned}$$

Thus, the mean hop number between nodes in a chain topology is $(N+1)/3$.

5 Example: The Ring Topology

In Figure 3 the *Ring* multi–node network topology is given:

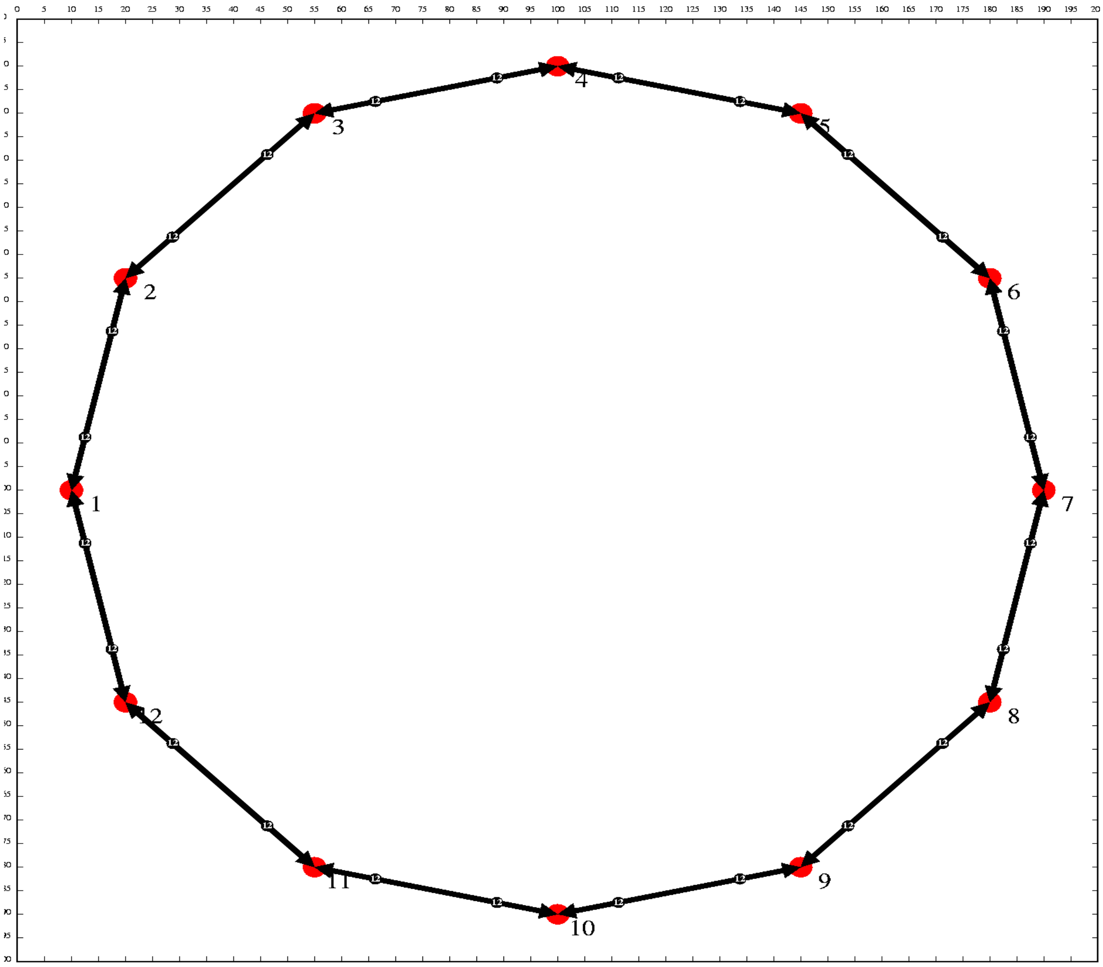

Figure 3: Topology of a Multi–Node Network.

Please, generate the adjacency (single–hop connectivity) matrix A and the connectivity Matrix C of the multi–node network.

1. Round

$$A^{(1)} = \begin{pmatrix} 0&1&0&0&0&0&0&0&0&0&0&1\\ 1&0&1&0&0&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&1\\ 1&0&0&0&0&0&0&0&0&0&1&0 \end{pmatrix}$$

$$B^{(1)} = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$C^{(1)} = \begin{pmatrix} 0&1&0&0&0&0&0&0&0&0&0&1\\ 1&0&1&0&0&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&0&0&0&0\\ 0&0&1&0&1&0&0&0&0&0&0&0\\ 0&0&0&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&1&0&1&0&0&0&0&0\\ 0&0&0&0&0&1&0&1&0&0&0&0\\ 0&0&0&0&0&0&1&0&1&0&0&0\\ 0&0&0&0&0&0&0&1&0&1&0&0\\ 0&0&0&0&0&0&0&0&1&0&1&0\\ 0&0&0&0&0&0&0&0&0&1&0&1\\ 1&0&0&0&0&0&0&0&0&0&1&0 \end{pmatrix}$$

2. Round

$$A^{(2)} = C^{(1)} \cdot C^{(1)} = \begin{pmatrix} 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 2 \end{pmatrix}$$

$$B^{(2)} = \begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \end{pmatrix}$$

$$C^{(2)} = C^{(1)} + B^{(2)} = \begin{pmatrix} 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 \\ 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 \\ 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 & 1 \\ 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \end{pmatrix}$$

3. Round

$$A^{(3)} = C^{(2)} \cdot C^{(2)} = \begin{pmatrix} 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 & 4 & 4 & 1 & 4 \\ 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 & 4 & 4 & 1 \\ 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 & 4 & 4 \\ 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 & 4 \\ 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 & 0 \\ 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 & 0 \\ 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 & 0 \\ 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 & 4 \\ 4 & 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 & 4 \\ 4 & 4 & 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 & 1 \\ 1 & 4 & 4 & 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 & 4 \\ 4 & 1 & 4 & 4 & 0 & 0 & 0 & 4 & 4 & 1 & 4 & 10 \end{pmatrix}$$

$$B^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 4 & 3 \\ 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 4 \\ 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 \\ 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 & 4 \\ 4 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 & 3 \\ 3 & 4 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 0 \\ 0 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(3)} = C^{(2)} + B^{(3)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 2 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 4 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 & 4 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 & 0 \\ 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 0 \\ 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 4 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 4 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 2 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 & 1 \\ 1 & 2 & 3 & 4 & 0 & 0 & 0 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

4. Round

$$A^{(4)} = C^{(3)} \cdot C^{(3)} = \begin{pmatrix} 60 & 40 & 23 & 12 & 26 & 44 & 50 & 44 & 26 & 12 & 23 & 40 \\ 40 & 60 & 40 & 23 & 12 & 26 & 44 & 50 & 44 & 26 & 12 & 23 \\ 23 & 40 & 60 & 40 & 23 & 12 & 26 & 44 & 50 & 44 & 26 & 12 \\ 12 & 23 & 40 & 60 & 40 & 23 & 12 & 26 & 44 & 50 & 44 & 26 \\ 26 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 26 & 44 & 50 & 44 \\ 44 & 26 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 26 & 44 & 50 \\ 50 & 44 & 26 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 26 & 44 \\ 44 & 50 & 44 & 26 & 12 & 23 & 40 & 60 & 40 & 23 & 12 & 26 \\ 26 & 44 & 50 & 44 & 26 & 12 & 23 & 40 & 60 & 40 & 23 & 12 \\ 12 & 26 & 44 & 50 & 44 & 26 & 12 & 23 & 40 & 60 & 40 & 23 \\ 23 & 12 & 26 & 44 & 50 & 44 & 26 & 12 & 23 & 40 & 60 & 40 \\ 40 & 23 & 12 & 26 & 44 & 50 & 44 & 26 & 12 & 23 & 40 & 60 \end{pmatrix}$$

$$B^{(4)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 5 & 6 & 5 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 5 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 & 5 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 6 \\ 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 5 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 5 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 6 & 5 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 5 & 6 & 5 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(4)} = C^{(3)} + B^{(4)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 \\ 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 \\ 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 \\ 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 \\ 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 & 5 \\ 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 & 6 \\ 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 & 5 \\ 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 & 4 \\ 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 & 3 \\ 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 & 2 \\ 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 & 1 \\ 1 & 2 & 3 & 4 & 5 & 6 & 5 & 4 & 3 & 2 & 1 & 0 \end{pmatrix}$$

Once again we are interested in the value for the mean number of hops within the multi–hop network. In contrast to the chain scenario the end points are now connected. This fact is taken into account by the two additional *1* in the adjacency matrix. To calculate the number of hops we see that even for the ring topology the connectivity matrix C is symmetric. The symmetry of the matrix is influenced by the an even or an odd number of nodes. This leads us to two different equations, Equation 16 and Equation 22 for an odd or an even number, respectively:

$$\overline{H_{odd}} = 2 \cdot \frac{1}{N(N-1)} \sum_{k=1}^{\frac{N-1}{2}} Nk \quad (16)$$

$$= \frac{2N}{N(N-1)} \sum_{k=1}^{\frac{N-1}{2}} k \quad (17)$$

$$= \frac{2}{N-1} \left(\frac{(\frac{N-1}{2}+1)\frac{N-1}{2}}{2} \right) \quad (18)$$

$$= \frac{1}{N-1} \left((\frac{N-1}{2}+1)\frac{N-1}{2} \right) \quad (19)$$

$$= \frac{1}{N-1} \left(\frac{(N-1)^2}{4} + \frac{N-1}{2} \right) \quad (20)$$

$$= \frac{N+1}{4} \quad (21)$$

$$\overline{H_{even}} = \frac{1}{N(N-1)} \left(\sum_{k=1}^{\frac{N-2}{2}} 2Nk + N \cdot N/2 \right) \quad (22)$$

$$= \frac{1}{(N-1)} \left(\sum_{k=1}^{\frac{N-2}{2}} 2k + N/2 \right) \quad (23)$$

$$= \frac{1}{(N-1)} \left(\frac{(\frac{N-2}{2}+1)\frac{N-2}{2}}{2} + N/2 \right) \quad (24)$$

$$= \frac{1}{(N-1)} \left(\frac{(N-2)^2}{4} + \frac{N-2}{2} + N/2 \right) \quad (25)$$

$$= \frac{1}{(N-1)} \frac{N^2-4N+4+2N-4+2N}{4} \quad (26)$$

$$= \frac{N^2}{4(N-1)} \quad (27)$$

Note,

$$\sum_{k=1}^{N} k = \frac{N(N+1)}{2} \quad (28)$$

A quick proof of the results can be obtained if a triangle (N=3) or a box (N=4) is assumed. For the triangle the mean hop number equals one and for the box it is 4/3.

Thus, the mean hop number between nodes in a ring topology with an odd number of nodes is $(N+1)/4$, while for a an even number of nodes the mean hop number is $\frac{N^2}{4(N-1)}$.

6 Example: The Mesh Topology

In Figure 4 the following multi–node network topology is given:

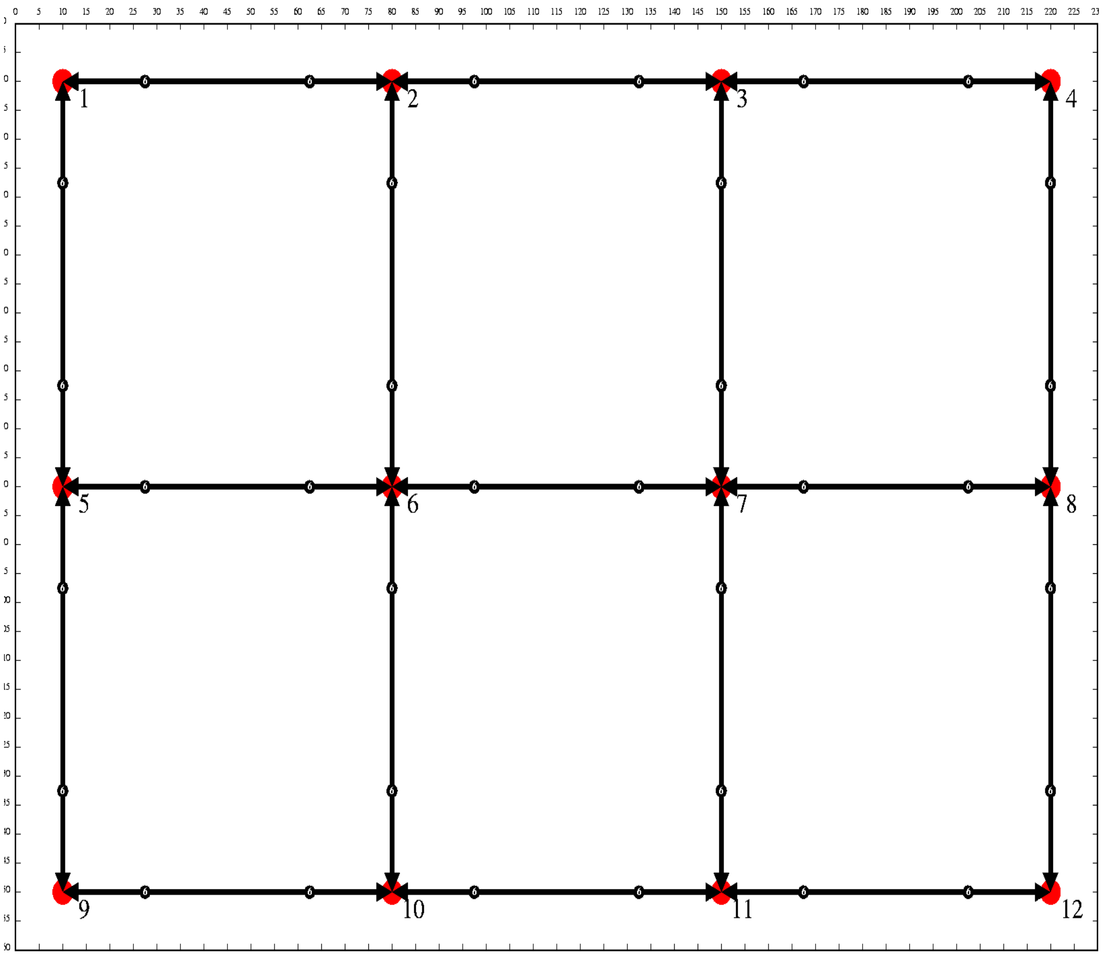

Figure 4: Topology of a Multi–Node Network.

The topology is referred to as a mesh network. A mesh network is characterized by M rows and N columns. The adjacency (single–hop connectivity) matrix A and the connectivity Matrix C of the mesh network is given in the following.

1. Round

$$
A^{(1)} = \begin{pmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

$$
B^{(1)} = \begin{pmatrix}
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0
\end{pmatrix}
$$

$$
C^{(1)} = \begin{pmatrix}
0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\
1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\
0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\
1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 1 & 0 \\
0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 \\
0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\
0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 & 0 \\
0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 1 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0
\end{pmatrix}
$$

2. Round

$$A^{(2)} = C^{(1)} \cdot C^{(1)} = \begin{pmatrix} 2 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 3 & 0 & 1 & 2 & 0 & 2 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 3 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 1 \\ 0 & 2 & 0 & 0 & 3 & 0 & 1 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 4 & 0 & 1 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 1 & 0 & 4 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 & 1 & 0 & 3 & 0 & 0 & 2 & 0 \\ 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 & 2 & 0 & 2 & 1 & 0 & 3 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 2 & 0 & 0 & 1 & 0 & 2 \end{pmatrix}$$

$$B^{(2)} = \begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 0 & 2 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 2 & 0 \\ 0 & 2 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 \\ 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 & 0 & 2 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 0 & 2 & 2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 \end{pmatrix}$$

$$C^{(2)} = C^{(1)} + B^{(2)} = \begin{pmatrix} 0 & 1 & 2 & 0 & 1 & 2 & 0 & 0 & 2 & 0 & 0 & 0 \\ 1 & 0 & 1 & 2 & 2 & 1 & 2 & 0 & 0 & 2 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 2 & 1 & 2 & 0 & 0 & 2 & 0 \\ 0 & 2 & 1 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 2 \\ 1 & 2 & 0 & 0 & 0 & 1 & 2 & 0 & 1 & 2 & 0 & 0 \\ 2 & 1 & 2 & 0 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 0 \\ 0 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 0 & 2 & 1 & 2 \\ 0 & 0 & 2 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 2 & 1 \\ 2 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 & 1 & 2 & 0 \\ 0 & 2 & 0 & 0 & 2 & 1 & 2 & 0 & 1 & 0 & 1 & 2 \\ 0 & 0 & 2 & 0 & 0 & 2 & 1 & 2 & 2 & 1 & 0 & 1 \\ 0 & 0 & 0 & 2 & 0 & 0 & 2 & 1 & 0 & 2 & 1 & 0 \end{pmatrix}$$

3. Round

$$A^{(3)} = C^{(2)} \cdot C^{(2)} = \begin{pmatrix} 14 & 6 & 5 & 4 & 6 & 10 & 8 & 8 & 5 & 8 & 12 & 0 \\ 6 & 19 & 8 & 5 & 10 & 10 & 14 & 8 & 8 & 9 & 8 & 12 \\ 5 & 8 & 19 & 6 & 8 & 14 & 10 & 10 & 12 & 8 & 9 & 8 \\ 4 & 5 & 6 & 14 & 8 & 8 & 10 & 6 & 0 & 12 & 8 & 5 \\ 6 & 10 & 8 & 8 & 15 & 10 & 9 & 4 & 6 & 10 & 8 & 8 \\ 10 & 10 & 14 & 8 & 10 & 24 & 12 & 9 & 10 & 10 & 14 & 8 \\ 8 & 14 & 10 & 10 & 9 & 12 & 24 & 10 & 8 & 14 & 10 & 10 \\ 8 & 8 & 10 & 6 & 4 & 9 & 10 & 15 & 8 & 8 & 10 & 6 \\ 5 & 8 & 12 & 0 & 6 & 10 & 8 & 8 & 14 & 6 & 5 & 4 \\ 8 & 9 & 8 & 12 & 10 & 10 & 14 & 8 & 6 & 19 & 8 & 5 \\ 12 & 8 & 9 & 8 & 8 & 14 & 10 & 10 & 5 & 8 & 19 & 6 \\ 0 & 12 & 8 & 5 & 8 & 8 & 10 & 6 & 4 & 5 & 6 & 14 \end{pmatrix}$$

$$B^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 3 & 0 & 0 & 3 & 4 & 0 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 3 & 4 \\ 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 4 & 3 & 0 & 3 \\ 3 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 0 & 4 & 3 & 0 \\ 0 & 0 & 3 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 3 & 4 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 & 0 \\ 4 & 3 & 0 & 0 & 3 & 0 & 0 & 0 & 4 & 3 & 0 & 0 \\ 0 & 3 & 4 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 3 \\ 3 & 0 & 3 & 4 & 0 & 0 & 0 & 3 & 0 & 0 & 0 & 0 \\ 4 & 3 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 4 & 3 & 0 & 4 & 3 & 0 & 0 & 3 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(3)} = C^{(2)} + B^{(3)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 & 2 & 3 & 4 & 0 \\ 1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 & 3 & 2 & 3 & 4 \\ 2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 & 4 & 3 & 2 & 3 \\ 3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 & 0 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 & 3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 \\ 2 & 3 & 4 & 0 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 \\ 3 & 2 & 3 & 4 & 2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 \\ 4 & 3 & 2 & 3 & 3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 \\ 0 & 4 & 3 & 2 & 4 & 3 & 2 & 1 & 3 & 2 & 1 & 0 \end{pmatrix}$$

4. Round

$$A^{(4)} = C^{(3)} \cdot C^{(3)} = \begin{pmatrix} 73 & 54 & 47 & 48 & 60 & 41 & 34 & 45 & 52 & 48 & 41 & 52 \\ 54 & 62 & 56 & 47 & 58 & 46 & 40 & 46 & 48 & 46 & 40 & 41 \\ 47 & 56 & 62 & 54 & 46 & 40 & 46 & 58 & 41 & 40 & 46 & 48 \\ 48 & 47 & 54 & 73 & 45 & 34 & 41 & 60 & 52 & 41 & 48 & 52 \\ 60 & 58 & 46 & 45 & 74 & 52 & 40 & 44 & 60 & 58 & 46 & 45 \\ 41 & 46 & 40 & 34 & 52 & 42 & 36 & 40 & 41 & 46 & 40 & 34 \\ 34 & 40 & 46 & 41 & 40 & 36 & 42 & 52 & 34 & 40 & 46 & 41 \\ 45 & 46 & 58 & 60 & 44 & 40 & 52 & 74 & 45 & 46 & 58 & 60 \\ 52 & 48 & 41 & 52 & 60 & 41 & 34 & 45 & 73 & 54 & 47 & 48 \\ 48 & 46 & 40 & 41 & 58 & 46 & 40 & 46 & 54 & 62 & 56 & 47 \\ 41 & 40 & 46 & 48 & 46 & 40 & 46 & 58 & 47 & 56 & 62 & 54 \\ 52 & 41 & 48 & 52 & 45 & 34 & 41 & 60 & 48 & 47 & 54 & 73 \end{pmatrix}$$

$$B^{(4)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 5 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(4)} = C^{(3)} + B^{(4)} = \begin{pmatrix} 0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 & 2 & 3 & 4 & 5 \\ 1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 & 3 & 2 & 3 & 4 \\ 2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 & 4 & 3 & 2 & 3 \\ 3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 & 5 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 & 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 & 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 & 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 & 3 & 2 & 1 & 0 & 4 & 3 & 2 & 1 \\ 2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & 0 & 1 & 2 & 3 \\ 3 & 2 & 3 & 4 & 2 & 1 & 2 & 3 & 1 & 0 & 1 & 2 \\ 4 & 3 & 2 & 3 & 3 & 2 & 1 & 2 & 2 & 1 & 0 & 1 \\ 5 & 4 & 3 & 2 & 4 & 3 & 2 & 1 & 3 & 2 & 1 & 0 \end{pmatrix}$$

Even for the more complex mesh network the mean hop distance can be calculated. Therefore we take a closer look at the connectivity Matrix C.

$$C^{(12x12)} = \begin{pmatrix} \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} & 1 & 2 & 3 & 4 & 2 & 3 & 4 & 5 \\ \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} & 2 & 1 & 2 & 3 & 3 & 2 & 3 & 4 \\ \mathbf{2} & \mathbf{1} & \mathbf{0} & \mathbf{1} & 3 & 2 & 1 & 2 & 4 & 3 & 2 & 3 \\ \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{0} & 4 & 3 & 2 & 1 & 5 & 4 & 3 & 2 \\ 1 & 2 & 3 & 4 & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} & 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} & 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 & \mathbf{2} & \mathbf{1} & \mathbf{0} & \mathbf{1} & 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{0} & 4 & 3 & 2 & 1 \\ 2 & 3 & 4 & 5 & 1 & 2 & 3 & 4 & \mathbf{0} & \mathbf{1} & \mathbf{2} & \mathbf{3} \\ 3 & 2 & 3 & 4 & 2 & 1 & 2 & 3 & \mathbf{1} & \mathbf{0} & \mathbf{1} & \mathbf{2} \\ 4 & 3 & 2 & 3 & 3 & 2 & 1 & 2 & \mathbf{2} & \mathbf{1} & \mathbf{0} & \mathbf{1} \\ 5 & 4 & 3 & 2 & 4 & 3 & 2 & 1 & \mathbf{3} & \mathbf{2} & \mathbf{1} & \mathbf{0} \end{pmatrix} \quad (29)$$

Therefore, we divide the $(N+M)x(N+M)$ Matrix into M^2 sub-matrixes SM with NxN elements, such that the connectivity matrix looks like:

$$C^{(N+M)x(N+M)} = \begin{pmatrix} SM_{1,1} & \dots & SM_{1,M} \\ \dots & \dots & \dots \\ SM_{1,M} & \dots & SM_{M,M} \end{pmatrix}, \quad (30)$$

where $SM_{i,j}$ for all $i = j$ is the same Matrix we calculated before in the chain example. For all other $SM_{i,j}$ with $i \neq j$ the sub–matrix $SM_{i,j}$ equals a sum out of the chain matrix and an additional part for all elements in this sub–matrix. As an example we look at $SM_{1,2}$:

$$SM_{1,2} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 2 & 3 \\ 3 & 2 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix} = \begin{pmatrix} 0 & 1 & 2 & 3 \\ 1 & 0 & 1 & 2 \\ 2 & 1 & 0 & 1 \\ 3 & 2 & 1 & 0 \end{pmatrix} + \begin{pmatrix} 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{pmatrix} \quad (31)$$

In this example for each element we added an one. For other sub–matrixes the first sub–matrix (known from the chain example) remains the same, but the additional value change. In general this value equals $|i - j|$, where $||$ stands for the absolute value.

In the following we refer to SM_C for the chain matrix and A_{i-j} for the matrix with the additional part, such that we can write:

$$C^{12x12} = \begin{pmatrix} SM_C & SM_C & SM_C \\ SM_C & SM_C & SM_C \\ SM_C & SM_C & SM_C \end{pmatrix} + \begin{pmatrix} 0 & A_1 & A_2 \\ A_1 & 0 & A_1 \\ A_2 & A_1 & 0 \end{pmatrix}, \quad (32)$$

Therefore, we find

$$C^{12x12} = \begin{pmatrix} SM_C & SM_C & SM_C \\ SM_C & SM_C & SM_C \\ SM_C & SM_C & SM_C \end{pmatrix} + \begin{pmatrix} 0 & 1 \cdot E & 2 \cdot E \\ 1 \cdot E & 0 & 1 \cdot E \\ 2 \cdot E & 1 \cdot E & 0 \end{pmatrix}. \quad (33)$$

$$C^{12x12} = \begin{pmatrix} SM_C & SM_C & SM_C \\ SM_C & SM_C & SM_C \\ SM_C & SM_C & SM_C \end{pmatrix} + E \cdot \begin{pmatrix} 0 & 1 & 2 \\ 1 & 0 & 1 \\ 2 & 1 & 0 \end{pmatrix}. \quad (34)$$

All the former work was necessary to simplify the following calculations. From Equation 34 we generate our general solution for the mean hop distance of the mesh network.

$$\overline{H} = \frac{M^2 SM_C + 2\sum_{k=1}^{M-1} k(M-k)E}{NM(NM-1)} \quad (35)$$

$$= \frac{M^2 \frac{N+1}{3} N(N-1) + 2N^2 \frac{M+1}{2\cdot 3} M(M-1)}{NM(NM-1)} \quad (36)$$

$$= \frac{M^2(N^2-1)N + N^2(M^2-1)M}{3NM(NM-1)} \quad (37)$$

$$= \frac{M(N^2-1) + N(M^2-1)}{3(NM-1)} \quad (38)$$

$$= \frac{N^2M - M + M^2N - N)}{3(NM-1)} \quad (39)$$

$$= \frac{N^2M - N + M^2N - M)}{3(NM-1)} \quad (40)$$

$$= \frac{N(NM-1) + M(MN-1)}{3(NM-1)} \quad (41)$$

$$= \frac{N+M}{3} \quad (42)$$

$$(43)$$

The chain topology is a sub set out of the mesh topology with M equal to 1, so Equation 35 becomes equal to Equation 11.

7 Example: The Cross Topology

In Figure 5 the following multi–node network topology is given:

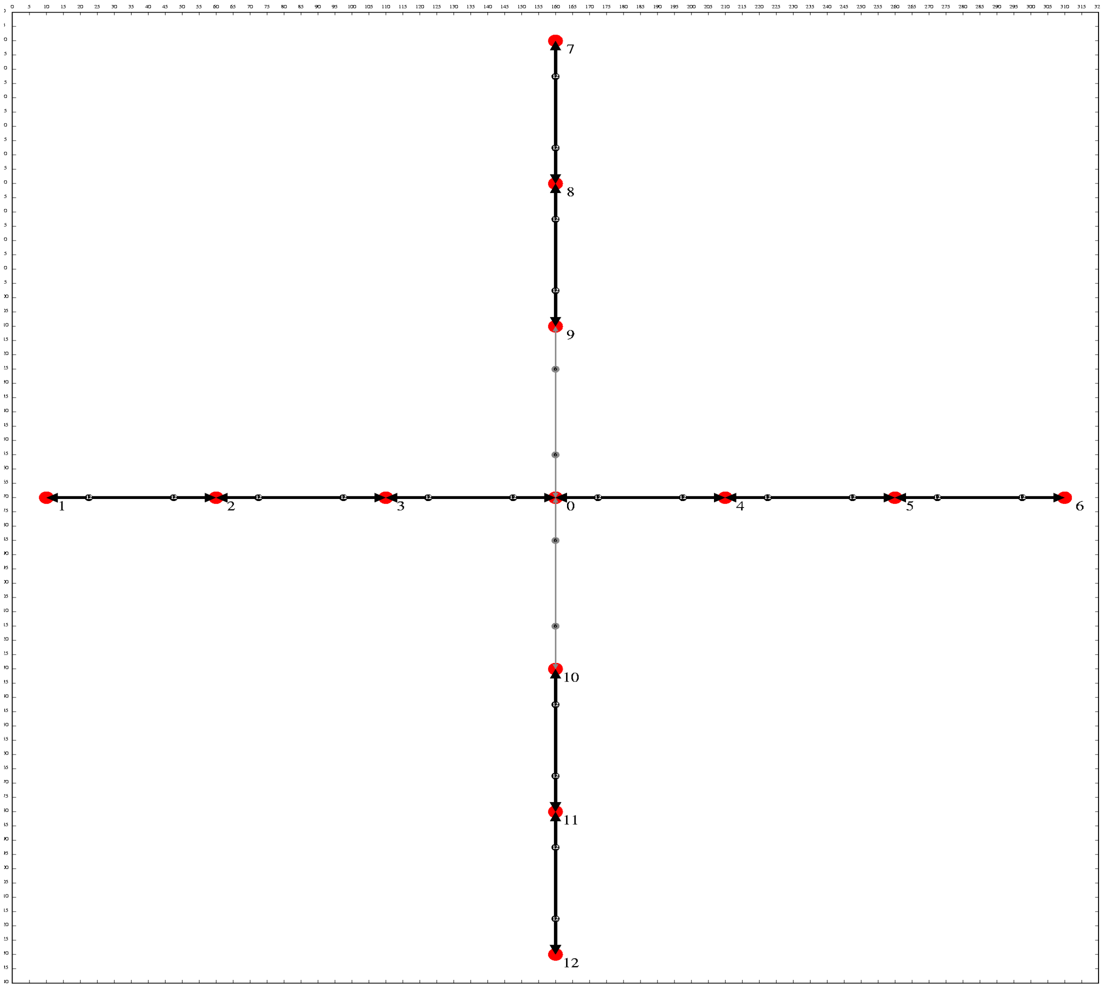

Figure 5: Topology of a Multi–Node Network.

Please, generate the adjacency (single–hop connectivity) matrix A and the connectivity Matrix C of the multi–node network.

1. Round

$$A^{(1)} = \begin{pmatrix} 0&0&0&1&1&0&0&0&0&1&1&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&1&0&0&0\\ 1&0&0&0&0&0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \end{pmatrix}$$

$$B^{(1)} = \begin{pmatrix} 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&0&0&0&0&0 \end{pmatrix}$$

$$C^{(1)} = \begin{pmatrix} 0&0&0&1&1&0&0&0&0&1&1&0&0\\ 0&0&1&0&0&0&0&0&0&0&0&0&0\\ 0&1&0&1&0&0&0&0&0&0&0&0&0\\ 1&0&1&0&0&0&0&0&0&0&0&0&0\\ 1&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&1&0&1&0&0&0&0&0&0\\ 0&0&0&0&0&1&0&0&0&0&0&0&0\\ 0&0&0&0&0&0&0&0&1&0&0&0&0\\ 0&0&0&0&0&0&0&1&0&1&0&0&0\\ 1&0&0&0&0&0&0&0&1&0&0&0&0\\ 1&0&0&0&0&0&0&0&0&0&0&1&0\\ 0&0&0&0&0&0&0&0&0&0&1&0&1\\ 0&0&0&0&0&0&0&0&0&0&0&1&0 \end{pmatrix}$$

2. Round

$$A^{(2)} = C^{(1)} \cdot C^{(1)} = \begin{pmatrix} 4 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 2 & 0 & 1 & 0 & 0 & 1 & 1 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 1 & 0 & 2 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 1 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 1 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \end{pmatrix}$$

$$B^{(2)} = \begin{pmatrix} 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 0 & 0 & 2 & 0 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 0 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 0 \end{pmatrix}$$

$$C^{(2)} = C^{(1)} + B^{(2)} = \begin{pmatrix} 0 & 0 & 2 & 1 & 1 & 2 & 0 & 0 & 2 & 1 & 1 & 2 & 0 \\ 0 & 0 & 1 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 2 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & 2 & 1 & 0 & 2 & 0 & 0 & 0 & 0 & 2 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & 0 & 1 & 2 & 0 & 0 & 2 & 2 & 0 & 0 \\ 2 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 2 & 0 & 0 & 0 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 0 & 0 & 0 \\ 1 & 0 & 0 & 2 & 2 & 0 & 0 & 2 & 1 & 0 & 2 & 0 & 0 \\ 1 & 0 & 0 & 2 & 2 & 0 & 0 & 0 & 0 & 2 & 0 & 1 & 2 \\ 2 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 1 & 0 \end{pmatrix}$$

3. Round

$$A^{(3)} = C^{(2)} \cdot C^{(2)} = \begin{pmatrix} 20 & 4 & 1 & 8 & 8 & 1 & 4 & 4 & 1 & 8 & 8 & 1 & 4 \\ 4 & 5 & 2 & 1 & 4 & 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 1 & 2 & 6 & 4 & 4 & 4 & 0 & 0 & 4 & 4 & 4 & 4 & 0 \\ 8 & 1 & 4 & 18 & 9 & 4 & 4 & 4 & 4 & 9 & 9 & 4 & 4 \\ 8 & 4 & 4 & 9 & 18 & 4 & 1 & 4 & 4 & 9 & 9 & 4 & 4 \\ 1 & 0 & 4 & 4 & 4 & 6 & 2 & 0 & 4 & 4 & 4 & 4 & 0 \\ 4 & 0 & 0 & 4 & 1 & 2 & 5 & 0 & 0 & 4 & 4 & 0 & 0 \\ 4 & 0 & 0 & 4 & 4 & 0 & 0 & 5 & 2 & 1 & 4 & 0 & 0 \\ 1 & 0 & 4 & 4 & 4 & 4 & 0 & 2 & 6 & 4 & 4 & 4 & 0 \\ 8 & 4 & 4 & 9 & 9 & 4 & 4 & 1 & 4 & 18 & 9 & 4 & 4 \\ 8 & 4 & 4 & 9 & 9 & 4 & 4 & 4 & 4 & 9 & 18 & 4 & 1 \\ 1 & 0 & 4 & 4 & 4 & 4 & 0 & 0 & 4 & 4 & 4 & 6 & 2 \\ 4 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 4 & 1 & 2 & 5 \end{pmatrix}$$

$$B^{(3)} = \begin{pmatrix} 0 & 3 & 0 & 0 & 0 & 0 & 3 & 3 & 0 & 0 & 0 & 0 & 3 \\ 3 & 0 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 4 & 0 & 0 & 4 & 3 & 3 & 4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 3 & 4 & 4 & 3 & 0 & 0 & 3 & 4 \\ 0 & 4 & 3 & 0 & 0 & 0 & 0 & 4 & 3 & 0 & 0 & 3 & 4 \\ 0 & 0 & 4 & 3 & 0 & 0 & 0 & 0 & 4 & 3 & 3 & 4 & 0 \\ 3 & 0 & 0 & 4 & 0 & 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 3 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 0 & 4 & 0 & 0 \\ 0 & 0 & 4 & 3 & 3 & 4 & 0 & 0 & 0 & 0 & 3 & 4 & 0 \\ 0 & 4 & 3 & 0 & 0 & 3 & 4 & 0 & 0 & 0 & 0 & 3 & 4 \\ 0 & 4 & 3 & 0 & 0 & 3 & 4 & 4 & 3 & 0 & 0 & 0 & 0 \\ 0 & 0 & 4 & 3 & 3 & 4 & 0 & 0 & 4 & 3 & 0 & 0 & 0 \\ 3 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 4 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(3)} = C^{(2)} + B^{(3)} = \begin{pmatrix} 0 & 3 & 2 & 1 & 1 & 2 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 & 4 & 0 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 2 & 1 & 0 & 1 & 3 & 4 & 0 & 0 & 4 & 3 & 3 & 4 & 0 \\ 1 & 2 & 1 & 0 & 2 & 3 & 4 & 4 & 3 & 2 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 & 0 & 1 & 2 & 4 & 3 & 2 & 2 & 3 & 4 \\ 2 & 0 & 4 & 3 & 1 & 0 & 1 & 0 & 4 & 3 & 3 & 4 & 0 \\ 3 & 0 & 0 & 4 & 2 & 1 & 0 & 0 & 0 & 4 & 4 & 0 & 0 \\ 3 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 1 & 2 & 4 & 0 & 0 \\ 2 & 0 & 4 & 3 & 3 & 4 & 0 & 1 & 0 & 1 & 3 & 4 & 0 \\ 1 & 4 & 3 & 2 & 2 & 3 & 4 & 2 & 1 & 0 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 & 2 & 3 & 4 & 4 & 3 & 2 & 0 & 1 & 2 \\ 2 & 0 & 4 & 3 & 3 & 4 & 0 & 0 & 4 & 3 & 1 & 0 & 1 \\ 3 & 0 & 0 & 4 & 4 & 0 & 0 & 0 & 0 & 4 & 2 & 1 & 0 \end{pmatrix}$$

4. Round

$$A^{(4)} = C^{(3)} \cdot C^{(3)} = \begin{pmatrix} 56 & 16 & 37 & 68 & 68 & 37 & 16 & 16 & 37 & 68 & 68 & 37 & 16 \\ 16 & 62 & 44 & 28 & 26 & 44 & 57 & 57 & 44 & 26 & 26 & 44 & 57 \\ 37 & 44 & 81 & 58 & 48 & 60 & 44 & 44 & 60 & 48 & 48 & 60 & 44 \\ 68 & 28 & 58 & 93 & 81 & 48 & 26 & 26 & 48 & 81 & 81 & 48 & 26 \\ 68 & 26 & 48 & 81 & 93 & 58 & 28 & 26 & 48 & 81 & 81 & 48 & 26 \\ 37 & 44 & 60 & 48 & 58 & 81 & 44 & 44 & 60 & 48 & 48 & 60 & 44 \\ 16 & 57 & 44 & 26 & 28 & 44 & 62 & 57 & 44 & 26 & 26 & 44 & 57 \\ 16 & 57 & 44 & 26 & 26 & 44 & 57 & 62 & 44 & 28 & 26 & 44 & 57 \\ 37 & 44 & 60 & 48 & 48 & 60 & 44 & 44 & 81 & 58 & 48 & 60 & 44 \\ 68 & 26 & 48 & 81 & 81 & 48 & 26 & 28 & 58 & 93 & 81 & 48 & 26 \\ 68 & 26 & 48 & 81 & 81 & 48 & 26 & 26 & 48 & 81 & 93 & 58 & 28 \\ 37 & 44 & 60 & 48 & 48 & 60 & 44 & 44 & 60 & 48 & 58 & 81 & 44 \\ 16 & 57 & 44 & 26 & 26 & 44 & 57 & 57 & 44 & 26 & 28 & 44 & 62 \end{pmatrix}$$

$$B^{(4)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 5 & 6 & 6 & 5 & 0 & 0 & 5 & 6 \\ 0 & 0 & 0 & 0 & 0 & 0 & 5 & 5 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 5 \\ 0 & 6 & 5 & 0 & 0 & 0 & 0 & 6 & 5 & 0 & 0 & 5 & 6 \\ 0 & 6 & 5 & 0 & 0 & 5 & 6 & 0 & 0 & 0 & 0 & 5 & 6 \\ 0 & 5 & 0 & 0 & 0 & 0 & 5 & 0 & 0 & 0 & 0 & 0 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 5 & 0 & 0 & 0 & 0 & 5 & 5 & 0 & 0 & 0 & 0 & 0 \\ 0 & 6 & 5 & 0 & 0 & 5 & 6 & 6 & 5 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(4)} = C^{(3)} + B^{(4)} = \begin{pmatrix} 0 & 3 & 2 & 1 & 1 & 2 & 3 & 3 & 2 & 1 & 1 & 2 & 3 \\ 3 & 0 & 1 & 2 & 4 & 5 & 6 & 6 & 5 & 4 & 4 & 5 & 6 \\ 2 & 1 & 0 & 1 & 3 & 4 & 5 & 5 & 4 & 3 & 3 & 4 & 5 \\ 1 & 2 & 1 & 0 & 2 & 3 & 4 & 4 & 3 & 2 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 & 0 & 1 & 2 & 4 & 3 & 2 & 2 & 3 & 4 \\ 2 & 5 & 4 & 3 & 1 & 0 & 1 & 5 & 4 & 3 & 3 & 4 & 5 \\ 3 & 6 & 5 & 4 & 2 & 1 & 0 & 6 & 5 & 4 & 4 & 5 & 6 \\ 3 & 6 & 5 & 4 & 4 & 5 & 6 & 0 & 1 & 2 & 4 & 5 & 6 \\ 2 & 5 & 4 & 3 & 3 & 4 & 5 & 1 & 0 & 1 & 3 & 4 & 5 \\ 1 & 4 & 3 & 2 & 2 & 3 & 4 & 2 & 1 & 0 & 2 & 3 & 4 \\ 1 & 4 & 3 & 2 & 2 & 3 & 4 & 4 & 3 & 2 & 0 & 1 & 2 \\ 2 & 5 & 4 & 3 & 3 & 4 & 5 & 5 & 4 & 3 & 1 & 0 & 1 \\ 3 & 6 & 5 & 4 & 4 & 5 & 6 & 6 & 5 & 4 & 2 & 1 & 0 \end{pmatrix}$$

8 Example: The Random Topology

In Figure 6 the following multi–node network topology is given:

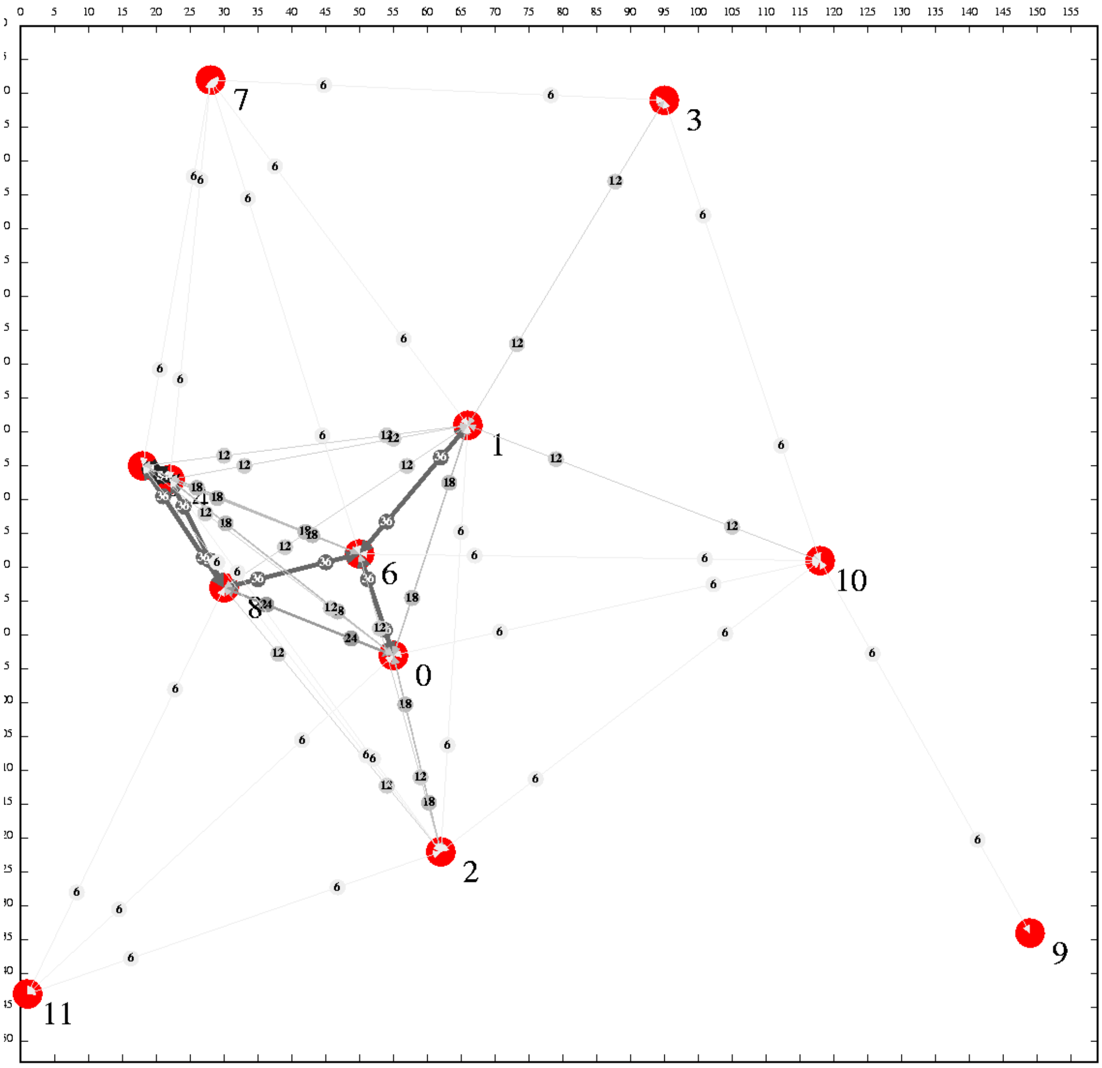

Figure 6: Topology of a Multi–Node Network.

Please, generate the adjacency (single–hop connectivity) matrix A and the connectivity Matrix C of the multi–node network.

1. Round

$$A^{(1)} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

$$B^{(1)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

$$C^{(1)} = \begin{pmatrix} 0 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 0 & 1 & 0 \\ 1 & 1 & 0 & 0 & 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 1 & 1 & 0 & 1 & 1 & 1 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix}$$

2. Round

$$A^{(2)} = C^{(1)} \cdot C^{(1)} = \begin{pmatrix} 8 & 6 & 7 & 2 & 5 & 5 & 6 & 4 & 6 & 1 & 3 & 2 \\ 6 & 9 & 6 & 2 & 6 & 6 & 7 & 4 & 5 & 1 & 4 & 3 \\ 7 & 6 & 8 & 2 & 5 & 5 & 6 & 4 & 6 & 1 & 3 & 2 \\ 2 & 2 & 2 & 3 & 2 & 2 & 3 & 1 & 1 & 1 & 1 & 0 \\ 5 & 6 & 5 & 2 & 7 & 6 & 6 & 3 & 5 & 0 & 4 & 3 \\ 5 & 6 & 5 & 2 & 6 & 7 & 6 & 3 & 5 & 0 & 4 & 3 \\ 6 & 7 & 6 & 3 & 6 & 6 & 8 & 3 & 5 & 1 & 3 & 3 \\ 4 & 4 & 4 & 1 & 3 & 3 & 3 & 5 & 4 & 0 & 3 & 0 \\ 6 & 5 & 6 & 1 & 5 & 5 & 5 & 4 & 7 & 0 & 4 & 2 \\ 1 & 1 & 1 & 1 & 0 & 0 & 1 & 0 & 0 & 1 & 0 & 0 \\ 3 & 4 & 3 & 1 & 4 & 4 & 3 & 3 & 4 & 0 & 6 & 2 \\ 2 & 3 & 2 & 0 & 3 & 3 & 3 & 0 & 2 & 0 & 2 & 3 \end{pmatrix}$$

$$B^{(2)} = \begin{pmatrix} 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 2 & 0 & 0 \\ 2 & 0 & 2 & 0 & 2 & 2 & 2 & 0 & 2 & 2 & 0 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 0 & 2 & 2 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 \\ 2 & 0 & 2 & 0 & 0 & 0 & 0 & 0 & 2 & 0 & 2 & 0 \\ 0 & 0 & 0 & 2 & 0 & 0 & 0 & 2 & 0 & 0 & 2 & 0 \\ 2 & 2 & 2 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 2 & 2 & 0 & 2 & 2 & 0 & 0 & 2 \\ 0 & 2 & 0 & 0 & 2 & 2 & 2 & 0 & 0 & 0 & 2 & 0 \end{pmatrix}$$

$$C^{(2)} = C^{(1)} + B^{(2)} = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 0 & 2 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 0 & 2 & 2 & 2 & 1 & 2 & 2 & 1 & 0 \\ 1 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 1 & 2 & 1 & 0 & 1 & 1 & 1 & 0 & 2 & 2 \\ 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 1 & 1 & 1 & 0 & 2 & 0 & 2 & 0 \\ 1 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 0 & 0 & 2 & 1 \\ 2 & 2 & 2 & 2 & 0 & 0 & 2 & 0 & 0 & 0 & 1 & 0 \\ 1 & 1 & 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 2 \\ 1 & 2 & 1 & 0 & 2 & 2 & 2 & 0 & 1 & 0 & 2 & 0 \end{pmatrix}$$

3. Round

$$A^{(3)} = C^{(2)} \cdot C^{(2)} = \begin{pmatrix} 20 & 16 & 19 & 18 & 15 & 15 & 18 & 12 & 16 & 11 & 19 & 12 \\ 16 & 17 & 16 & 18 & 14 & 14 & 17 & 12 & 13 & 9 & 18 & 11 \\ 19 & 16 & 20 & 18 & 15 & 15 & 18 & 12 & 16 & 11 & 19 & 12 \\ 18 & 18 & 18 & 31 & 14 & 14 & 17 & 21 & 15 & 15 & 23 & 22 \\ 15 & 14 & 15 & 14 & 19 & 18 & 16 & 15 & 17 & 14 & 16 & 13 \\ 15 & 14 & 15 & 14 & 18 & 19 & 16 & 15 & 17 & 14 & 16 & 13 \\ 18 & 17 & 18 & 17 & 16 & 16 & 20 & 13 & 15 & 11 & 19 & 11 \\ 12 & 12 & 12 & 21 & 15 & 15 & 13 & 21 & 14 & 16 & 15 & 18 \\ 16 & 13 & 16 & 15 & 17 & 17 & 15 & 14 & 19 & 14 & 16 & 14 \\ 11 & 9 & 11 & 15 & 14 & 14 & 11 & 16 & 14 & 21 & 10 & 14 \\ 19 & 18 & 19 & 23 & 16 & 16 & 19 & 15 & 16 & 10 & 26 & 16 \\ 12 & 11 & 12 & 22 & 13 & 13 & 11 & 18 & 14 & 14 & 16 & 23 \end{pmatrix}$$

$$B^{(3)} = \begin{pmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 3 & 3 & 0 & 3 & 3 & 0 & 0 & 3 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 3 & 0 & 0 & 0 & 3 & 0 & 3 & 0 & 0 \end{pmatrix}$$

$$C^{(3)} = C^{(2)} + B^{(3)} = \begin{pmatrix} 0 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 1 & 0 & 1 & 1 & 1 & 1 & 1 & 1 & 1 & 2 & 1 & 2 \\ 1 & 1 & 0 & 2 & 1 & 1 & 1 & 2 & 1 & 2 & 1 & 1 \\ 2 & 1 & 2 & 0 & 2 & 2 & 2 & 1 & 2 & 2 & 1 & 3 \\ 1 & 1 & 1 & 2 & 0 & 1 & 1 & 1 & 1 & 3 & 2 & 2 \\ 1 & 1 & 1 & 2 & 1 & 0 & 1 & 1 & 1 & 3 & 2 & 2 \\ 1 & 1 & 1 & 2 & 1 & 1 & 0 & 1 & 1 & 2 & 1 & 2 \\ 2 & 1 & 2 & 1 & 1 & 1 & 1 & 0 & 2 & 3 & 2 & 3 \\ 1 & 1 & 1 & 2 & 1 & 1 & 1 & 2 & 0 & 3 & 2 & 1 \\ 2 & 2 & 2 & 2 & 3 & 3 & 2 & 3 & 3 & 0 & 1 & 3 \\ 1 & 1 & 1 & 1 & 2 & 2 & 1 & 2 & 2 & 1 & 0 & 2 \\ 1 & 2 & 1 & 3 & 2 & 2 & 2 & 3 & 1 & 3 & 2 & 0 \end{pmatrix}$$

9 Summary

Table 1: Mean hop number of different multi–hop network topologies.

topology	maximum hop number	mean hop number
chain	N-1	$\frac{N+1}{3}$
ring	$\left[\frac{N}{2}\right]$	$\frac{N+1}{4}$ $\frac{N^2}{4(N-1)}$
mesh	N+M-2	$\frac{N+M}{3}$

10 Acknowledgement

When we started to write this manual, we were not aware of the splendid work by L. E. Miller. Parts of this work were inspired by the work of L. E. Miller. To give the students the possibility to do further studies on the material of L. E. Miller is used more or less the same notation. His work can be found at [1]. Note, the calculation of the connectivity presented in [2] differs slightly from the calculation presented here. Of course both results in the same connectivity matrix, but the presented approach needs less calculation rounds, which helps to reduce the amount of processing power.

References

[1] L. E. Miller. Catalog of network connectivity models. http://w3.antd.nist.gov/wctg/netanal/netanal_netmodels.html, April 2001. 36

[2] L. E. Miller. Multihop connectivity of arbitrary networks. http://w3.antd.nist.gov/wctg/netanal/ConCalc.pdf, March 2001. 36